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Solvency II facing the short-termism barrier

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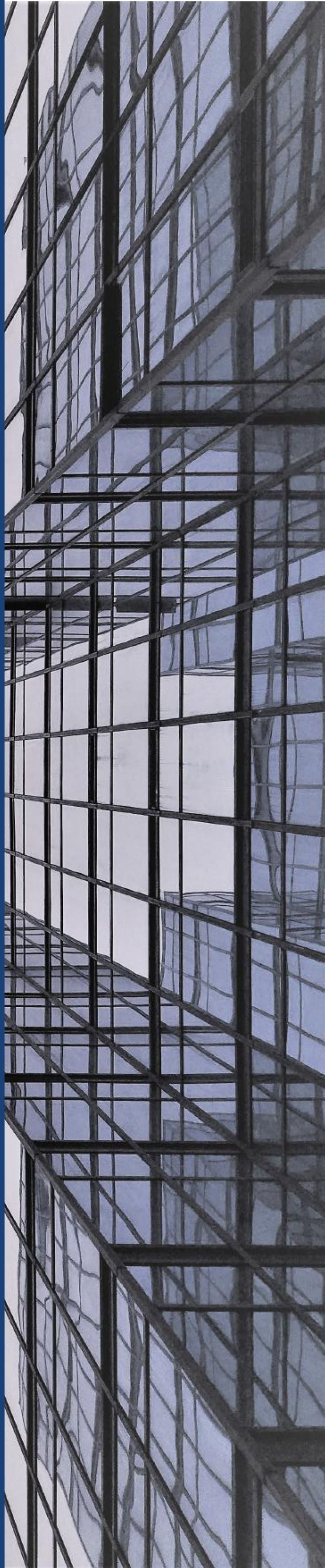

This technical note was written on May 10, 2021 by Christian Walter[1] at the request of Af2i, the Association française des investisseurs institutionnels (French Association of Institutional Investors). This association strongly supports the goals of the 2030 Agenda for sustainable finance and wish to mainstream them into their professional and regulatory tools.

This technical note is based on academic papers all previously published in peer-reviewed journals or chapters of scientific books. The academic papers of Ch. Walter have been re-used and rearranged or updated, in the spirit of the question asked by the AF2i.

In order not to encumber the text with systematic references to C. Walter's academic papers, the list of these papers is given in the Appendix. The other academic references mentioned in the technical note are listed at the end in the "References" section.

Abstract

This technical note is a piece for contributing to the sustainable European stake in order to interlock financial systems with the objectives of the 2030 Agenda. It is intended to be used as a platform for discussion between risk management practitioners in the financial industry and the regulator, as well as operators and scholars who have already applied investigative methods in other disciplines. Using a new approach, one aims to shed a light on what the morphology of randomness does to sustainable projects. It is argued that any sustainable finance project devised to fit the Sustainable Development Goals (SDGs) needs a paradigm shift in the morphology of randomness underlying financial risk modelling. It is argued that Leibniz's principle of continuity and Quetelet's theory of averaging are pervasive in Solvency II and that they ground a culture of risk in finance, acting as a mental model that prevents finance experts and practitioners from taking a long-term view, rendering Solvency II misaligned with the long-term targets of sustainability.

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Content

Introduction: sustainable financial risk modelling **5**
A cognitive perspective **7**
From mathematical models to mental models in finance 7
The principle of continuity from physics to finance 8
The principle of continuity as a mental model in finance 9
The principle of continuity at work in financial models **12**
Path-continuity in financial modelling: time-risk equivalency 12
Path-continuity in financial techniques: risk disappears 14
Path-continuity at its zenith: the third quantification convention 16
Path-continuity in regulatory framework: the time-scaling of risk 19
Revisiting the short-termism issue **22**
Two competitive risk cultures 22
Responsible regulation and financial black swans 23
Conclusion **25**
References **26**
Appendix **28**

Introduction: sustainable financial risk modelling

1. For several years, a huge wave of willingness to convert financial activities in the direction of sustainable finance taking into account the long term perspectives as well as environmental and societal issues (the UN's SDGs) has been emerging in Europe. An important step was accomplished in 2017 when the European Commission set up a high-level expert group (HLEG) to define a strategic action plan for the green transition of European finance and the financing of sustainable growth. This group has delivered a final report and a set of recommendations[2] that have led to the European Sustainable Finance Action Plan in 2018[3], which aims to connect finance with sustainability.

2. The recommendations of HLEG include ten key actions that can be divided into three categories: reorienting capital flows towards a more sustainable economy, including sustainability into risk management and fostering transparency and long-termism[4]. Indeed, this preoccupation with strengthening long-termism is becoming necessary at a time when more and more studies and reports are highlighting a trend towards short-termism in the investments of the main financial players traditionally present in the long-term segment. However, everyone easily understands that taking the SDGs into account is only possible in a long-term perspective. As HLEG stressed, "sustainability cannot develop in a context where investment is dominated by short-term considerations"[5]. This is the reason why the European Commission has launched a "call for advice" to each of the European Supervisory Authorities (ESAs) in order to have a clear understanding of the potential barriers to the long term perspectives and in particular of the barrier raised by short-termism: "the ESAs are expected to assess the extent to which short-termism is present and can be considered problematic"[6]. On 1 February 2019, the Commission requested advice from ESMA, EBA and EIOPA on undue short-term pressure from the financial sector on corporations. The recommendation of the three supervisors was to better include ESG criteria in the investment process design.

3. This technical note is a piece for contributing to the abovementioned stake in order to interlock financial systems with the objectives of the 2030 Agenda. It is intended to be used as a platform for discussion between investors, asset owners and asset managers, risk managers in the financial industry and the regulator, as well as operators and scholars who have already applied investigative methods in other disciplines. It is a proposition which presents a new diagnostic for sustainable regulation, using a cognitive approach about the morphology of randomness. To put it differently, this technical note aims to shed a light on what the morphology of randomness does to sustainable projects.

4. We propose to complete the existing answers with a new cognitive angle. Based on existing work, this technical note uses some of the arguments already published in research articles[7], with the aim of outlining something new, which does not seem to be addressed in the existing answers to short-termism, which is: any sustainable finance

[2] High-Level Expert Group (2018)
[3] European Commission (2018)
[4] European Commission (2020)
[5] High-Level Expert Group (2018), p. 45.
[6] European Commission (2019)
[7] See ref. at the end of the paper.

project devised to fit the Sustainable Development Goals (SDGs) needs a paradigm shift in the morphology of randomness underlying financial risk modelling, by integrating the characteristics of "nature" and sustainability into the modelling carried out. It is argued that the main problem with neoclassical finance risk modelling mainstreamed by Solvency II is its underlying morphology of randomness. This morphology creates a short-term oriented risk culture because of the embeddedness of short-term horizon in the morphology of randomness. It is argued that Leibniz's principle of continuity and Quetelet's theory of average are pervasive in Solvency II and ground a culture of risk in finance acting as a mental model obstructing the long-term view for finance experts and practitioners. In a nutshell, we relate the notion of sustainability to the morphology of randomness.

5. The outline of this note is as follow. Section 1 presents the cognitive perspective and introduces the principle of continuity. Section 2 develops the effects of the principle of continuity in financial techniques, financial practices and financial regulation. Section 3 revisits the issue of short-termism by shedding light on the consequences of the principle of continuity and claims for a new culture of risk based on a new morphology of randomness to foster sustainable projects in finance and sustainable regulation.

A cognitive perspective

6. As noticed by Ippoliti (2017, p. 121) “rules, laws, institutions, regulators, the behaviour and the psychology of traders and investors are the key elements to the understanding of finance, and stock markets in particular”. In this vein, the short-termism can be thought as the result of human actions and financial rules and institutions.

7. The notion of short-termism has been the focus of several definitions in the studies undertaken. For example in HLEG (2018, p. 45) short-termism is described as the tendency to prioritise near-term shareholder interests and profitability at the expense of the long-term growth of the firm. In most situations, short termism is understood in a behavioural way. For example, “in the EIOPA report, “short-term behaviour has not to be confused with short investment horizon. It is a focus on short-term profits without ensuring sufficient investment for long-term needs and development”[8]. According to EIOPA, there are three short-termism drivers: short-term stock prices; high-speed computer trading and reduced trading times and transaction costs; shorter reporting frequencies which damage long-term strategies and this is done by trying to rely on a particular approach to short-termism, mixing behavioural analysis and portfolio structure analysis.

8. One would like to propose another approach to short-termism, based on the notion of shared mental model, acting as mental disposition for practitioners, academics and regulators. To do this, it proposes to make a detour via philosophy of science in order to investigate the shared mental model as applied to the global framework of risk modelling in Solvency II.

From mathematical models to mental models in finance

9. According to Wikipedia, a mental model is an explanation of a person’s thought process about how something works in the real world[9]. For example, a runner participating in a mountain race on steep paths mentally represents to himself the characteristics of a risk (falling stones, tripping over a precipice, dangerous parts of the path, etc.) and uses that representation to anticipate the outcome of an action (running faster, slowing down, etc.). Mental models shape actors’ normative issues. Shared mental models lead to the emergence of norms and standards as in the case of the Solvency II directive, which results from a shared mental model on the morphology of randomness.

10. Mantzavinos (2001) considers that, whatever their activity or intellectual aspirations, people must first ensure their material existence and, in order to do so, they must have mental models to act on. For Mantzavinos, one way to get out of the intellectual impasses of primitive mental models is to look at the side of science. It seems that all scientific disciplines come to the conclusion that there are no normative “facts” independent of the minds of the people who think them and whose interaction generates social norms. No

[8] EIOPA (2019), p. 8.
[9] The notion of mental model is precisely defined in Craik (1943), Johnson-Laird and Byrne (2002), Mantzavinos (2001).

phenomenon is beyond mental activity, whether individual or shared. Therefore, the question arises as to how to think of the problem-solving activity with the mental activity.

11. One of the answers is to consider that we can have a first approximation of mental representations by mathematical models. Actually, a mathematical model is a rational mental conception which, beginning from an initial idea, builds a representation of reality – dynamic or static – which constitutes therefore a mental tool. This mental tool shapes the structure of the agents' expectations in front of the environment. A mental tool can induce an information filter causing selective perception, a marker of which has been cognitive bias on statistical tests. A canonical example of a mental model producing a cognitive bias in the analysis of statistical financial data is given by the article by Granger and Orr (1972) in which the authors truncate the extreme values of the empirical distribution to make the statistical results fit the Gaussian distribution (De Bruin and Walter, 2017).

12. Mathematical models of financial risk have buried epistemological premises. These epistemological foundations are related to a specific morphology of randomness that produces a specific risk culture. To put it another way, above and beyond the technical choices of the components of financial risk modelling, a philosophical background has existed throughout the 20th century in academic research in finance. Thus, risk modelling issues raise problems or puzzles that are usually addressed by philosophy of science. It is the reason why the methodological proposal of the paper is that philosophy of science can serve as a fundamental tool for clarifying and understanding the context of mental model acting in financial risk modelling.

13. It is for this reason that these debates, far from being limited to academic concerns within a small circle of specialists in the philosophy of science, who would remain distant from the financial stakes of risk modelling, and who would have no impact on concrete financial practices, are on the contrary the matrix of divergent professional positions. The mindset behind financial risk modelling can stage entirely different views on what was important to capture and how to model it.

14. In summary, it is argued that the epistemological background of financial risk modelling has acted as a "mental model" for financial market professionals, academics and regulators designing the Solvency II directive. It is also argued that this epistemological background is mainly based on the principle of continuity.

The principle of continuity from physics to finance

15. The principle of continuity – change is continuous –, over and above its use in technical devices, is a principle from natural philosophy postulating that in nature, things change gradually rather than suddenly. The most compact expression is found in the famous Latin saying *Natura non facit saltus* (meaning "nature does not make jumps"), which we owe to Leibniz. This principle can be understood either as mathematical or metaphysical.

16. The principle of continuity was the source of differential and integral calculus as performed by Leibniz, then Newton. It provided the foundations for the ideas of Carl von Linné on classification of species, and later Charles Darwin on the theory of evolution

and Alfred Marshall in economics. For our purpose, it is interesting to read today what Marshall wrote in 1890 in his *Principles of Economics*: (1890): "If the book has any special character of its own, that may perhaps be said to lie in the prominence which it gives to this and other applications of the *Principle of Continuity*".

17. Very early on, comments challenged Marshall's attempt to impose continuity as a basis for economic modelling. As early as 1927, the principle of continuity was attacked by the Cambridge Italian economist Piero Sraffa (Martins, 2013). In the 1960s, the principle of continuity began to be challenged. In 1966, Norbert Wiener observed, "just as primitive peoples adopt the Western modes of denationationalized clothing and of parliamentarism out of a vague feeling that these magic rites and vestments will at once put them abreast of modern culture and technique, so *the economists have developed the habit of dressing up their rather imprecise ideas in the language of the infinitesimal calculus*" (Wiener, 1966, p. 89, 90, emphasis added). Next, Wiener emphasized: "here some recent work of Mandelbrot is much to the point. He has shown the intimate way in which the commodity market is both theoretically and practically subject to random fluctuations arriving from the very contemplation of its own irregularities is something much wilder and much deeper than has been supposed, and that the *usual continuous approximations to the dynamics of the market must be applied with much more caution* than has usually been the case, or not at all" (Wiener, 1966, p. 92, emphasis added).

18. What is interesting about Wiener's comment is that he does so from knowledge of Benoît Mandelbrot's work. Indeed, Mandelbrot was the first to assert that continuity was dangerous for financial risk modelling, and all his scientific work was a development of how to take discontinuities into account, from the first articles of 1962 and 1963, the same ones that Wiener had noticed. For Mandelbrot, the principle of continuity created a "smooth" mental model for finance in which no breaks could occur, a universe without risk. To this he contrasted a "rough" mental model, in which any stock market path was inherently irregular, even at the smallest scales. The discontinuity does not disappear with the downscaling, and one can consider the large discontinuities as an outgrowth of the small discontinuities and vice versa.

19. In the twentieth century, the development of science, physics and genetics, made it necessary to question the principle of continuity. Quantum mechanics postulated discrete (i.e. discontinuous) energy levels, and genetics took discontinuities into account. Yet economics, from which neoclassical financial theory originated, remained outside of these important intellectual transformations. Despite much evidence of economic phenomena that could not be explained by continuity, this principle remained in force until the end of the 20th century and until now. Everything was going on as if, for mysterious reasons that had nothing to do with science, the principle of continuity was considered "natural" for finance and, in any case, preferable to discontinuous approaches.

The principle of continuity as a mental model in finance

20. It is argued that both finance professionals (bankers, traders, investors, regulators, analysts, "quants" etc.) and research professionals (academics in finance, financial mathematics, mathematical finance, and financial actuaries) have been strongly

influenced by an almost systematic use of the "principle of continuity". Its financial mathematical expression is the continuous Brownian representation of financial risk. To put it differently, one argues that the Brownian representation of financial risk acted as a mental model. An epistemic framework on risk measurement has resulted from a shared mental model on uncertainty, based on the continuous Brownian representation in financial risk modelling.

21. This continuous Brownian representation has become so entrenched in our ways of thinking about risk and risk prevention that makes it a set of options become self-evident and no longer need to be proved or questioned, that are no longer challenged by the problems experienced, as we will explain below. A cognitive phenomenon prevented financial experts and academics from using more accurate mathematical models, relieved from the continuity or Gaussian assumption, as exemplified in Grander and Orr (1972).

22. This denial of alternative financial risk models that are nevertheless more successful in terms of practical results resists the usual epistemological analysis and represents an "epistemological puzzle" in this respect (Walter, 2019). It is for this cognitive reason that it is argued that the puzzle of maintaining the continuity assumption in financial risk modelling despite numerous statistical and practical invalidations could be illuminated by reference to the use of a mental model.

23. The issue of choosing this mental model over another, more effective one, is a matter of what is called epistemic ethics. Epistemic ethics is a way of updating virtue ethics in the field of mental choices. Epistemic ethics shows that we are responsible for our mental choices with regard to the consequences of these choices. Thus, for example, if we want to take into account the objectives of sustainability or the long term perspectives, not all mental choices are equivalent. Some mental choices are "ethical" in this sense, and others are not. Among the intellectual virtues are humility and prudence. Humility means the acceptance of statistical results even when they contradict the assumptions of mathematical models. Prudence refers to a consciousness of what is right.

24. This issue was raised long ago by the French physicist and historian of science Pierre Duhem. As Duhem noticed, we prefer models with simple formulas that allow for calculations and elegant theoretical constructions. In *The Aim and Structure of Physical Theory* (1906), Duhem questioned the role of models in science. He puts forwards the following statement on the relationship between models and accidents: "We shall remind industrialists, who have no care for the accuracy of a formula provided it is convenient, that the simple but false equation sooner or later becomes, by an unexpected act of revenge of logic, the undertaking which fails, the dike which bursts, the bridge which crashes; *it is financial ruin when it is not the sinister reaper of human lives*" (Duhem, 1906, p. 93, emphasis added).

25. The Leibnizian philosophy of continuous randomness has acted as a "philosophical anchor" on practitioners, academics and regulators and in particular inside the Solvency II framework. The choice of this "philosophical anchor" is a result of historical financial developments, the "making of finance" (Chambost et al. 2019), because of – one could imagine with Duhem – its ease of application and the possibility of developing a very sophisticated mathematical apparatus to address a legitimate question. This mental model makes the culture of financial risk embedded in Solvency II incompatible with the

integration of the sustainability criterion because of the morphology of the randomness that constitutes it.

26. To conclude this part, it is argued that the short-termism is the visible result of a troublesome seminal intellectual choice of continuous randomness, itself the outcome of the principle of continuity. A major consequence of the continuity principle in finance is, as we will see in the next section, to make the time span vanish. The principle of continuity puts the agent in front of an intellectual "wall" that makes it blind to everything that cannot be reproduced by its model, in particular what we can call an uncertain future. Consequently, there is no room for the notion of long term perspectives.

The principle of continuity at work in financial models

27. Let us now turn to specify how the principle of continuity is at work in financial modelling, which will allow us to introduce the debate between continuity and discontinuity in financial modelling approaches. How does the continuity principle come into effect in the framework of financial modelling? In this section, we elaborate on this topic.

Path-continuity in financial modelling: time-risk equivalency

28. Stochastic processes are an important component in contemporary financial modelling of the market dynamic of asset prices. In order to see more clearly the implications of the adoption of the notion of continuity of price changes without introducing complicated mathematics, we present a minimal mathematical formalism that will allow us to clarify our argument

29. Let us fix some notations. In what follows, the price of any security at time t is denoted by $S(t)$ (S for Stock, Security or Share). The "simple return" on this security corresponds to real monetary gains or losses. The gain (or loss) is the difference $S(t) - S(0)$, and the "natural" arithmetic return is given by the basic formula $(S(t) - S(0)) / S(0)$. Academics and practitioners (traders, risk managers, etc.) are generally interested in the continuous rate of return between time 0 and time t (the continuous compound return). This quantity is denoted:

$$X(t) = \ln S(t) - \ln S(0) \quad (1)$$

In (1), "ln" is the natural logarithm. The move from the natural return to the logarithmic return represents the logarithmic convention used in return calculations in financial modelling. Financial computations are usually performed using $X(t)$. In the financial modelling literature, the quantity $X(t)$ is conceived as a *stochastic process* which describes the return dynamic. The definition (1) means that prices evolve according to the equation:

$$S(t) = S(0) \exp X(t) \qquad (2)$$

In (2), "exp" is the exponential function. The randomness hypothesis in financial modelling assumes that $X(t)$ is a stochastic process. For example, during almost a century, the main hypothesis of financial modelling was that $X(t)$ is a random walk.

30. *The standard model of price variations* is now presented in a single picture. The most commonly used continuous-time stochastic process in finance is Brownian motion, one of the best known Lévy processes, which are stochastic processes with stationary independent increments[10]. The Brownian representation of the cumulative return dynamic $X(t)$ has been based on Brownian motion since the seminal works of Louis Bachelier

[10] Lévy processes, labelled after the French mathematician Paul Lévy, are continuous-time stochastic processes with independent and identically distributed (IID) increments. With the exception of Brownian motion with drift, they consist entirely of jumps. See for example Bertoin (1998) and Sato (1999).

(Bachelier, 1900) and Maury Osborne (Osborne, 1959). In this specific representation, the return dynamic of any financial asset at time *t* is given by an equation associating a return "trend" and the "risk" of the given asset:

$$X(t) = mt + \sigma W(t) \qquad (3)$$

31. In this equation, *m* is the mean parameter which gives the trend growth (proportional to time *t*) of the cumulative return and □ is the standard deviation parameter (square root of variance, diffusion coefficient of Brownian motion), termed "volatility" in the markets, designed to capture the *scale* of the distribution of possible returns around the mean (± 5% or ± 50%), the degree of uncertainty regarding future returns. As a result, with the standard[11] Wiener process $W(t)$, "risk" is considered as a random excursion around the trend, and the shape and trajectory are given by the Wiener process, scaled by the value of the standard deviation. To put it differently, in the standard model, the volatility of a financial asset is considered a proxy for risk. This is a *one-dimensional assessment* of risk: the scale of risk. The *morphology* of risk (the shape of the risk profile), which is supposed to capture the uncertainty of future returns, is described by the standard Wiener process.

32. Moving from asset returns to asset prices, the equation for the price dynamic is:

$$S(t) = S(0)\exp\left(mt + \sigma W(t)\right) \qquad (4)$$

Equation (4) defines an exponential Brownian motion. This equation became the standard model of market dynamics in 1965 when it was recommended by Paul Samuelson.

33. Because the financial quantities described by the standard model are the mean (average trend of return fluctuations) and the volatility, we get a representation of the mean-variance map associated with Brownian dynamics. This is the mean-variance map of the standard model representation of the risk-return analysis. This map contains all the first-generation elementary information about the market dynamics of returns. Hence, the mathematics of the standard model of cumulative returns is clearly linked to the Brownian representation.

34. One main feature of the Brownian representation is its combination of the two characteristics of path-continuity in the return dynamics and normality in return distributions. The continuity property reflects a liquid market, where there are many buyers and sellers for a given security. The normality property reflects the risk of a given security: returns are more frequently close to the mean return than far from the mean return. Another feature of the Brownian representation comes from the equivalence between risk and time. In the Brownian framework, □^{2} = *t* meaning that variance is a linear function of the length of the time. This equivalency between risk and time has the effect of making time (or risk) "disappear" in a deterministic perspective. Risk can be eliminated by the passage of time, or time can be compressed into instantaneous risk. This interchangeability creates a mental disposition that does not include the time span in investment perspectives.

35. In fact, one of the counterintuitive consequences of this continuous framework is the disappearance of risk for management purposes. The mathematicians of finance, basing

[11] "Standard" means $W_0 = 1$, the increments are independent and for $0 < s < t$, the increment $W(t) - W(s)$ is normally distributed with mean zero and variance $t\text{-}s$

their work on assumptions of an idealized market with a continuous Brownian representation of randomness, have shown that for any fixed amount at a given maturity (payment of an insurance claim, a guaranteed amount, etc.), it is possible to entirely tame risk, whatever the degree to which the risk on the relevant phenomenon (financial market, real economy, demographics, climate change, etc.) materializes, because of the type of randomness chosen.

36. Since financial risk seems tamed, the principle of continuity leads to an apparently risk-free economy. And a risk-free economy turns into an economy in which limits are no longer justified and, therefore, an economy without limits. An economy without limits is an economy is likely to become unsustainable. This time-risk equivalency was manifested in the design of financial instruments and risk management by techniques that incorporated the principle of continuity into the daily life of professionals. We now turn to these techniques.

Path-continuity in financial techniques: risk disappears

37. The principle of continuity subsequently trickled down into all of neoclassical economic thought, which was the source of contemporary finance. The principle of continuity permeated all neoclassical economic models, which was the source of neoclassical finance theory. It was at the heart of the probabilistic assumptions in financial risk modelling and in this respect the financial risk modelling is an application of this continuity principle.

38. The financial theory mathematically modelled since 1952 came in the wake of this principle of continuity. The principle of continuity was the mental model that governed researchers' intuition in the mathematical writing of financial risks, in their research work, and then in their teaching of finance. The principle of continuity thus became the cornerstone of a representation of the probable in practical finance, which contained methods of reasoning for professional practices derived from financial mathematics based on the same principle. The principle of continuity contained methods of reasoning for financial practitioners derived from risk models based on the continuity assumption and, in this sense, the principle of continuity was at the core of a large number of financial techniques.

39. Path-continuity and normality of returns form the common core of the portfolio theory, option pricing theory, and fundamental asset pricing methods. Portfolio theory is considered a central factor in the making of finance into a scientific discipline (MacKenzie, 2006). It emerged between 1950 and 1965, that is, between the validation of the random walk model and the introduction of the efficient market hypothesis in asset management practices. Portfolio theory was first developed by Harry Markowitz in 1952, then refined by James Tobin in 1958, and achieved its classic twofold formulation from William Sharpe in 1963 with respect to the linear probability model and in 1964 with respect to the equilibrium model.

40. This latest theoretical development had a deep impact on the portfolio management profession: it gave portfolio managers an incentive to aim for maximum diversification of assets and extensive indexation on so-called "benchmarks": market indices created to

consist of securities representing some aspect of the total market (such as the S&P 500). This widespread indexation has been both the norm and the limitation of the asset management industry, in theory and in practice too. It is closely linked to the Brownian representation of return dynamics because of Quetelet's view about averages (Walter, 1996). It was labelled this influence "Quetelet's influence" on asset management methodology (Walter, 2005). In introducing the idea of the optimal mean-variance portfolio, Markowitz and Sharpe used and thus validated the continuous Brownian representation.

41. To solve the problem of optimizing portfolios, it was necessary to hypothesize a stochastic process for the time series of securities' returns on the market, since calculation of the variance-covariance matrix requires a probability characterization of all the comovements by these securities, in the form of a probability vector concerning the whole market: a joint distribution of returns for all securities. This distribution was a joint multinormal distribution. Assuming multinormality in securities' price changes made it possible for the calculations to be made, and for MV-optimal portfolios to be designed. Therefore, the foundations for the quantitative approach to investment management come from the continuous Brownian representation of price variations.

42. A few years later came the birth and development of the option pricing theory. This consolidated the key role of continuous Brownian motion in the financial industry. The development of option pricing tools became so important in finance in the 1970's and 1980's, with intensive use of second-order diffusion processes that it was impossible to question the use of continuous Brownian motion in finance. The formulae developed by Fisher Black, Myron Scholes and Robert Merton in 1973, and later, the fundamental theorem of asset pricing of Harrison, Kreps and Pliska in 1979 and 1981, assume continuous price change. As MacKenzie and Spears (2014, p. 401) state: "It is the strategy of Black-Scholes modelling writ large: Find a perfect hedge, a continuously-adjusted portfolio of more basic securities that will have the same payoff as the derivative, whatever happens to the price of the underlying asset" (emphasis added). With a mental representation built on continuity, financial risk logically disappears since if things change gradually and steadily, their development is always predictable and safeguards can be found in techniques of financial derivatives, which are all based on the principle of continuity.

43. Practical application of these ideas to build financial models – which will then be used to value assets and make decisions – requires construction of what is termed a "replicating portfolio". The replicating portfolio is a portfolio which shares the same properties as the asset it replicates (e.g. series of cash flows or terminal value). The replication technique can be used to hedge or value any type of asset, especially derivatives. This breakthrough in mathematical financial techniques paved the way for an invasion of the "real" economy by derivatives. The pillar of this technique needs "market-consistent" valuation, whose visible mathematical trace is the risk-neutral probability.

44. The AOA principle derives from the continuity principle and represents the intellectual cornerstone of the dominant contemporary financial approaches. Based on the pioneering mathematical results of Harrison and Kreps (1979) and Harrison and Pliska (1981) under the AOA assumption, mathematical finance has come to consider it possible to extract expected returns on investments from market prices. In these conditions, market prices are considered the perfect measure of discounted expected cash flows and can be used to

"reveal" an underlying risk-neutral probability (see below), unique all tradable securities, uncertainty being governed by what Mandelbrot termed "mild" randomness (i.e. fully describable by continuous Brownian representation).

45. To move from market prices to expected returns, assumptions must be made about the rate of return. In this approach, the risk-free rate of return is used as the expected rate of return for investors. Changing the discount rate is equivalent to changing the numeraire of the asset (a little like an exchange rate can be used to express a value in a different currency). But this change also means that real-world probabilities are replaced by a new probability termed the "risk-neutral probability". For calculative purposes the "continuous finance" has imagined a new world, the risk-neutral world, in which all invested assets are assumed to provide the same expected rate of return, namely the risk-free rate, regardless of the risk of each specific asset. This purely mathematical transformation has major financial virtues. Notably, it neutralizes a form of variability in the discount rate, which now becomes the same for all assets, risky or otherwise, a situation that was impossible without the risk-neutral technique.

46. This AOA has played a central role in finance. It is amazing how much can be deduced from this one simple financial assumption. Practitioners in various sectors of finance have subscribed to this assumption to be able to use this new "risk-neutral" technology, which has paved the way for the full financialisation of the global economy. The powerful elegance of the "AOA continuous randomness" representation for market-consistent valuation is a major development which has profoundly transformed financial practices over the last thirty years. Hence, the introduction of fair value valuation in regulatory valuation has led the regulator (through the Solvency II Directive) to develop a valuation system for insurance companies based on financial market practices, the so-called "market consistency".

47. This principle was still predominant in the 1990s despite the emerging evidence of extreme values in the tails of empirical distributions. At the end of the 20th century, many financial techniques such as portfolio insurance or the calculation of capital requirements in the insurance industry still assumed that (financial) nature does not make jumps, and therefore promoted continuity.

48. It has been said that neoclassical economic theory had constructed a mathematical utopia of the market (Chen, 2017; Lawson, 1997, 2003). It is argued that this mathematical utopia was due to the use of the continuity principle. Hence, to shed a light on Lawson's statement, "Contemporary academic economics is not in a healthy state" (Lawson, 1997, p. 3) and to answer the question asked by Chen, "What's wrong with economic math?" (Chen, 2017, p. 17), one answers: the presupposition of the principle of continuity forming the Brownian representation of finance and financial risk.

Path-continuity at its zenith: the third quantification convention

49. It is possible to consider three periods in turn, showing how the principle of continuity coupled with the use of the average has progressively transformed professional financial practices. It has been shown that three financial "quantification conventions" have organized the history of financial thought (Chiapello and Walter, 2016). In particular, we

showed that the discount rate has been transformed by the emergence of the third quantification convention in the sense of short-termism. We now look more closely at this point.

50. The first financial quantification convention is the “actuarial discounting convention”. With this convention, the present value is determined through a simple calculation: known cash flows were discounted to present value using a constant interest rate. Both the numerators (cash flows) and the discount factor (the inverse of the discount rate) are deterministic. It is considered that there is no uncertainty affecting future cash flows or the discount rate. The discount rate used introduces into financial valuation a powerful simplification that is not obvious in itself: the same rate is used for all maturities of cash flows, such that the remuneration on money is considered identical for every maturity, whether one day or one year. In other words, the yield curve is flat. While some idea of risk is empirically taken into account by the choice of a higher or lower discount rate, this risk is not based on a statistical calculation.

51. These are points that change with the second financial quantification convention, the “mean-variance convention”. With this the risk is defined by the variance (or its square root, the standard deviation). As a result of this, the level of risk premium is determined using the Capital Asset Pricing Model (CAPM) devised by Sharpe (1964). This model gives the risk premium level (using the linear relationship of the beta coefficient). In this financial quantification convention, the discount factor becomes variable, as it depends on the beta coefficient, but is not random.

52. The second financial quantification convention introduces a new and extremely important idea for the financial practices: the relevant discount rate for calculating a present value is related to the rate of return on a specific portfolio known as the “mean-variance-optimal tangent” portfolio: this portfolio has been considered equivalent to the “market” since the seminal paper by Sharpe, and this “market” needs a proxy representation in order to apply this theoretical research to make practical real-life decisions. Serving as proxies is precisely the function of market indexes (such as the Dow Jones Industrial Average indexes). Apart from the technicity of this change, the new development is that financial valuation is now associated with market equilibrium. In the second quantification convention, valuation of any item requires a mean-variance optimal tangent portfolio, which in practice means actors must keep up with an index. And conversely, any mean-variance optimal tangent portfolio (or market index) becomes a possible instrument for asset valuation.

53. The third financial quantification convention, the “market-consistent convention”, extends this idea. The Solvency II directive is explicitly built on this third quantification. The discount factor, which in the second convention only varied with the investments studied (i.e. the risk specific to each one, measured by the beta), has now become random. “Stochastic discounting” replaces traditional discounting, whether the rate used is given (with the first convention) or results from an equilibrium model such as the CAPM (in the second convention). The stochastic discount factor is termed the “deflator”, just as a traditional operation deflates nominal values to real values.

Quantification convention	Characteristic of the discount factor
1	Constant
2	Variable: deterministic
3	Variable: stochastic

Table 1: The three stages of the mathematical form of the discount factor

54. The third financial quantification convention completely reshapes financial theory, with its cornerstone concept of "absence of arbitrage opportunity" (AOA) in an arbitrage-free market. With this extremely strong concept, valuations of investments become "market-consistent" and pave the way for extended use of "fair market value" (FMV) as defined by international accounting standards.

55. Let us summarize our point. While the key operational concept of the 1960s was the mean-variance optimal portfolio, leading to implementation of risk-return analysis in the asset management industry, the key operational concept of the 1980s was this new idea of replication with AOA principle, leading to implementation of risk-neutral analysis in the derivatives industry. Given the importance of the risk-neutral property of arbitraged prices, for instance to calculate the present value of any asset with a market-consistent framework, this feature can be considered as both the cornerstone and the mark of the third quantification convention.

56. The change in quantification convention is, as just seen, always supported by developments in financial theory, particularly the invention of new mathematical models which make all sorts of values calculable because they are founded on very restrictive assumptions. The first convention is rooted in calculation of DCF, which proposes a mathematical form that can make very different investments commensurable: all are treated as sums paid out with a view to receiving monetarily quantified returns in the future. The second is based on a reduction of the universe of investments under the two criteria of mean (the return) and variance (volatility as a measure of risk) which makes portfolio management models possible. Finally, the third convention is built on a new mathematical expression that has facilitated the rise of derivatives.

57. These mathematical models have been introduced into management instruments that govern financial decisions and help to shape professional practices. In each period, it is the models with the most easily-handled mathematical forms that are incorporated into calculation systems and accompany the transformation in the professions of finance. The most reassuring branches of finance, because they are the most readily translatable into calculation machines, are the ones that have spread to the point of becoming the dominant forms.

58. New professions have arisen while others have been changed. Practices previously considered highly risky because they involved a kind of gambling have seen particularly impressive expansion since the new calculation methods appeared to make them calculable and optimizable, and therefore controllable and manageable. Advances in modelling, combined with the increasingly massive collection of data and rising calculation capacities, mean that in finance, as elsewhere, people are able to undertake actions every day that used to be considered risky or impossible.

59. Thanks to the techniques of derivatives based on the principle of continuity, each component of the risk can now be covered by creation of ad hoc instruments that can be traded on a market; this proliferation of financial instruments and derivatives markets triggered extensive change in the international capital markets, which have become a widespread risk market because all is risk, since it can be reduced to standardized risk. In parallel, the banks and insurance companies, whose job used to be to bear long-term risks in their balance sheets, have learned to pass those risks by simply selling them or by securitizing them. By the grace of the models of the third financial quantification convention, all assets (credits) and all liabilities (insurance commitments) can now be sold on a market under fair price terms. This is precisely the property of market-consistent valuation models to be able to price such brand new assets.

60. At a certain point, this mental model became a consensus shared by all. The strength of the consensus on the relevance of this convention is such that its models are promoted by banking and insurance regulators (with the Basel II framework for banks and the Solvency II framework for insurance). Ultimately it looks as though the regulators, took on the idea initially advocated by the ISDA that good risk management could be carried out by well-informed financial actors practicing daily valuation of their risk exposure based on market prices. This validated Alan Greenspan's remark that, for all the brain power of civil servants, they were unable to master their business better than the professionals. This is what the third convention's mathematical instruments propose.

61. And so these instruments have also overseen a *general disqualification of traditional risk assessment methods*, which used to be based on ad hoc analyses. Since bankers can rapidly pass on the risks they acquire through lending, they no longer need to know their clients. All they need is a statistical approach to the default risk by category of borrowers. Insurers, meanwhile, are gradually *abandoning the traditional risk estimation methods that until now constituted their expertise*. It is true that Solvency II mentions the notion of the prudent man. But the prudence discussed in Solvency II is restricted by a conservative approach. Whereas prudence as a virtue in the sense of Greek antiquity meant sensitivity to what is just. By mentally adhering to the principle of continuity, the "prudent man" has lost his sensitivity to what is right and has replaced it with fear of risk. This loss of sensitivity to what is right has led the most experienced professionals to lose their sense of judgment and to become imprudent. To use a medical metaphor, it was as if professionals had lost their natural defences to risk perception and had become "immunodeficient" to risk. This immunodeficiency disorder could be viewed as the effect of a "mental virus" that caused a "contagion" or "pandemic" in the financial markets. Since this intellectual pandemic conveys a continuous Brownian representation of price changes, it could be termed a "Brownian virus".

Path-continuity in regulatory framework: the time-scaling of risk

62. The principle of continuity also underlies prudential regulation as a key implicit element of regulatory frameworks (Basel II, Solvency II). Actually, these regulations require banks and insurance companies to hold sufficient assets to buffer their risks. Continuous Brownian motion entails time scaling of risk in the sense that one given horizon (e.g., t) of a return distribution is scaled to another (e.g., $t \times a$). This means that the distribution

of X ($t \times a$) is the same as the distribution of X (t)$\times \sqrt{a}$. This is called the *scaling property* of Brownian motion or the *square-root-of-time rule* of scaling for risk-based approaches.

63. This scaling property leads to scaling of volatility (risk) in the sense that the volatility at scale t multiplied by the length of duration a is equal to the volatility at scale t multiplied by the square-root-of-time duration, i.e. □ (t)$\times \sqrt{a}$. The square-root-of-time rule is widely used in the Basel II and Solvency II regulations which promote the calculation and implementation of a probabilistic measure of risk quantity named “Value-at-Risk” (hereafter VaR). The minimum capital requirement is an estimated quantile of a return distribution (10-day 95% VaR metrics). The 10-day VaR is obtained by applying time scaling of risk using the square-root-of-time rule: VaR (10-day) = VaR (1-day)$\times \sqrt{10}$. This relationship is not “natural” but results from strong assumptions about the price process: its Brownian continuity. This situation corresponds to a continuous limit of price changes. In this case, X (t) is a continuous stochastic process.

64. We now elaborate on these assumptions. To be able to calculate a VaR, it is necessary to firstly define the variability structure of the price change process. To be able to define the increments of $X(t)$, it is, too, necessary to choose a characteristic “size” of the increments (one day, one week, one month etc.), denoted by the Greek letter □. The variable □ is the characteristic scale of the increments (daily scale, weekly scale etc.). Once this scale is chosen, it is possible to consider the increments of X (t), which are periodical returns (one-day return, one-week return etc.) of the following form:

$$X(t) - X(t-\tau) \stackrel{\text{def}}{=} Z(t,\tau) \qquad (5)$$

The random variable Z (t, □) is the law of the increments of X (t) for the scale □. This notation is straightforward, but it uses a notation convention that deserves to be emphasized. We keep visible □ to highlight the fact that the structure of price changes is strongly dependent on the scale □. Hence the calculation of the VaR on random variable Z (t, □) is defined at a given scale □. In the Basle III and Solvency II rules, one calculates VaR for □□=10 days and the square-root-of-time rule is assumed to work:

$$\text{VaR}(Z, 10 \text{ days}) = \sqrt{10} \times \text{VaR}(Z, 1 \text{ day}) \qquad (6)$$

The practical “square-root-of-time-rule” states that the annual VaR is obtained by the monthly VaR multiplied by the square root of the duration measured in months, i.e., 12. Etc.

65. The square-root-of-time-rule is a mathematical consequence of the scale invariance of Brownian motion. In fact Brownian motion is a self-similar (fractal) process B (t) such as:

$$B(a \times t) \equiv \sqrt{a} \times B(t) \qquad 6($$

where the symbol ≡ indicates an equality in distribution. The scaling exponent of Brownian motion is 0,5 from which it follows the square-root-of-time-rule of VaR. The above equation means that the shape of the distribution of returns is invariant when the time scale is changed by the square root of time. The scale invariance of Brownian motion is a core concept of the VaR metrics and calculations.

65. The square-root-of-time rule underlying the regulatory requirements for calculating minimum capital is a very narrow subset of the time-scaling rule of risk, and is directly

attributable to the assumption that return dynamics can be modelled by the continuous Brownian representation of price dynamics. This scaling property of the Brownian representation also promotes the widespread practice of calculating annualized volatility from weekly volatility: based on the weekly percentage change, the annualized volatility is equal to the weekly volatility (the standard deviation of the data) multiplied by $\sqrt{52}$.

66. This example illustrates how far the continuous Brownian representation has penetrated the calculation routines of financial practitioners and regulators, even if they do not know its theoretical foundations.

Revisiting the short-termism issue

67. Having presented the principle of continuity, we are now in a position to revisit the issue of short-termism with a different approach.

Two competitive risk cultures

68. There are many discrepancies between continuous representations and empirical financial phenomena, summarized by the "non-Brownian syndrome" (Walter, 2013, p. 262). A large number of practical difficulties have been encountered in financial applications of Brownian representation in the tools of practical finance. A large number of "stylized empirical facts" have emerged from statistical analysis of price variations in financial markets (for reviews see for example Cont, 2001; Sewell, 2011). The term "stylized fact" is used to refer to empirical findings that are ubiquitous over time and consistent across markets (e.g., heavy tails, intermittency, volatility clustering, etc.). These stylized facts invalidate many of the implications resulting from the Brownian representation of price dynamics. A "view from outside" on finance (Ippoliti, 2017) strongly supports general properties and representation of stock markets behaviour that cannot be explained with reference to Brownian representation of price dynamics.

69. Let us now consider the random variable representing the law of increments at scale □, namely *Z* (*t*,□□). Two alternatives exist when □ approaches to zero (infinitesimal scale). Either *Z* is approaching to zero, or not. In the first case (infinitesimal price change), we say that the price changes have a continuous limit. Then we are in the context of the continuity of economic and financial variables. In the second case, no. The discontinuities do not disappear at the limit. This is the main debate between competitive mental models: continuity or discontinuity of price changes.

70. The issue of market jumps is an old one in financial modelling. If there is any topic of scientific discussion that runs through almost the entire 20th century in financial modelling, it is certainly the shape of the distribution of *Z* (*t*,□□). It has been well documented in the academic literature that, in general, the empirical distribution of *Z* (*t*,□□) has thick tails, too thick to be Gaussian[12]. This story has been termed a "tale of fat tails" (Walter 2015, p. 467). The fat tails of empirical distributions of price changes have become an established stylized fact in the literature. In a rough universe, the distribution of *Z* (*t*,□□) is non-Gaussian at all scales □. The roughness is pervasive. In most cases, we imagine that price discontinuity refers to the existence of jumps. We emphasize here that discontinuity refers to roughness.

71. The shift from this risk culture to another risk culture is an important issue for the implementation of any theory of sustainable finance and ecological finance. Indeed, a financial risk model designed to be sustainable must take into account events or consequences that cannot be evaluated today and therefore cannot be integrated into a

[12] A good introduction to heavy tailed distributions in finance is the handbook edited by Svetlozar Rachev (2003).

model where everything is evaluated in a definite way according to the risk. This model must also be realistic, i.e. it must take into account the real characteristics of risk in the economy and the environment.

72. That continuity is a disabling property for financial modelling and that processes with discontinuities have to be implemented is something that is well known today in current research in finance, especially in financial risk modelling. Many mathematical models of financial risk are built on the basis of discontinuities (Aït-Sahalia et al., 2009; Barndorff-Nielsen, 1997; Bouchaud et al., 1998; Boudt et al., 2011; Eberlein and Prause, 1998; Kou, 2002; Liu and Hong, 2011; Wang and Tan, 2013 among others; see Walter 2017 for taxonomy). However, this innovation is relatively recent, dating at most from the end of the 20th century and early 2000s. The Solvency II revision could represent an opportunity to update the theoretical background knowledge base in regulation, and to move to a fully discontinuous approach to financial dynamics.

73. To conclude, it is argued that short-termism is the effect of a troublesome risk culture, itself coming from a troublesome philosophy of continuous randomness, itself the outcome of the principle of continuity. Thus the principle of continuity can be thought of as a philosophy anchor that causes the risk culture to be unsustainable. The risk culture associated with the continuity principle is not sustainable because of its morphology of randomness.

74. A good example of the need for changing the risk culture is made of the prudential regulation rules. In his 2018 report, the HLEG claims that "the application of the 'Think Sustainability First' approach should help to clarify what kind of legislation is most suitable to stimulate sustainable finance" (HLEG 2018, p. 62). We call "responsible regulation" a regulation that thinks "sustainability first".

Responsible regulation and financial black swans

75. Responsible regulation is at least as important as responsible banking or responsible investment, and sustainable regulation is at least as important as sustainable insurance.

76. It is known that multilateral institutions have acknowledged the need for a profound reform of the global financial system with emergence of Principles for Responsible Banking (PRB), Principles for Sustainable Insurance (PSI) and Principles for Responsible Investment (PRI).

77. However, the prudential regulations established after the 2008 crisis have had unexpected effects, just as dangerous as the ones they sought to address. Official reports have shown that international regulations have pro-cyclical effects and create potentially more dangerous financial situations than those that prevailed before the regulations were put in place. For example, it is mentioned that "the mark-to-market accounting rules for assets held in long-term portfolios" or some other aspects of prudential regulation can be procyclical. According to the EIOPA 2019 report, "the use of market values can be seen

a source of pressure for short-termism"[13]. More generally, the HLEG (2018, p. 48) report introduces the idea that the short-term behaviours could result from the regulation itself.

78. One of the consequences of the use of continuous Brownian representation is the inability to deal with the problem of discontinuities and the resulting intellectual cleavage in the appreciation of complex situations. A very good example of this inability to properly understand what is at stake is given by Alan Greenspan's editorial (16/03/08) in the *Financial Times* about the 2008 financial crisis: "We will never be able to anticipate all discontinuities in financial markets". Greenspan cannot imagine discontinuities being incorporated into a probabilistic description of the market. For Greenspan, (financial) nature does not make a leap and discontinuities are just unthinkable.

79. How to consider the discontinuities? The current research in finance dealing with financial risk is well aware of discontinuity and that there are many articles which try to overcome this issue. However, the status of discontinuities in continuous Brownian risk culture is thought of as an "extra-feature" of the stochastic process, not as the core characteristic of the process itself. This way of thinking about financial dynamics divides market movements into two regimes. The first is a "normal" regime in which variations are continuous. The second is a crisis regime in which a market breakdown occurs. With this mental model, discontinuities are assimilated to "outliers", relegating rare events such as 1987 crash to this dustbin category. For Mandelbrot, a very important issue was to include the so-called "outliers" as not outliers. For example, using a pure jump process as a Lévy process, the statistical properties of these "outliers" are identical to the statistical properties of small fluctuations.

80. For Nassim Taleb (2009), discontinuities are unpredictable exogenous "black swans". But it has been shown (Le Courtois et al. 2020) that financial regulation resulting from continuous Brownian representation could create endogenous financial black swans. This is because the risk culture of continuous Brownian finance can create synchronisation of financial risk management practices, leading to the "robotisation" of financial activities (Rodarie, 2007).

81. This technical note argues that it is not useful to isolate large variations from small ones. Moreover, it is argued that this separation encourages investors to take a short-term view because, with only the smooth fluctuations in view, and considering that the large fluctuations are black swans, they are led to disregard the long-term management of the large risks.

82. We have to consider that "black swans" (large discontinuities) are not different from the smallest variations (small discontinuities). A challenge to risk modelling is to consider that all discontinuities are the result of the same market phenomenon that allows price formation. Consequently, the separation made between the two regimes, normal and extreme, is not based on any physical reality.

[13] EIOPA (2019), p. 51.

Conclusion

83. In its 2018 report, the HLEG asked that consideration be given to "how Solvency II could be adapted to further facilitate long-term investments while maintaining a strong risk-based nature" (HLEG, 2018, p.72). The present technical note answers this to request by saying: by implementing sustainable financial risk modelling. This risk modelling will be sustainable if it removes the continuity principle from the epistemological background of the risk models used in the technical contents of the directive. To put it differently, "think sustainability first" implies "think discontinuity first".

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Appendix

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